Hard-Decision in COVQ Over Waveform Channels

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Abstract

A channel optimized vector quantizer (COVQ) is studied for the case of transmission over waveform channel. In this work, a number of modulation schemes with multidimensional signal constellations are considered, specifically, results on the binary signaling, M-ary phase-shift keying (MPSK) and M-ary quadrature amplitude modulation (MQAM) performance using COVQ over a waveform channel are provided. The proposed system, based on COVQ with hard-decision decoding, is optimized for additive white Gaussian noise (AWGN) and flat-fading Rayleigh channels. In addition, when flat-fading Rayleigh channel is assumed, diversity techniques are used and evaluated to improve the performance of the system.

1 Introduction

For source encoding, it is well known that Vector Quantization (VQ) ([1]) is an asymptotically optimal technique in the rate-distortion sense. Added to this desirable and theoretical property, nowadays, VQ is becoming a mature technique used for speech and images coding. Nevertheless, due to omnipresent channel errors, specially in fading environment, it is of great interest to design robust and optimized VQ coding systems against channel noise. Traditionally, the source coding and channel coding procedures are performed separately. Such as separation can be optimal in the limit of infinite delay, which is naturally not possible in practise due to obvious reasons. Performing a join source-channel coding we can achieve good performance at moderate delay and complexity. COVQ systems ([2], [3], [4]) carry out a join source-channel coding. The mentioned systems are optimized for the case of discrete memoryless channels (DMC). In [5] a joint source-channel coder is designed with the particularity that the encoder, the modulator and the decoder are jointly optimized for a Gaussian channel. More recently, the work [6] evaluates a soft-decision COVQ system, using binary signaling as modulation technique, optimized for Rayleigh-fading channels.

In this work we extend COVQ system study to a multidimensional signal constellation, several modulation schemes (binary signaling, MPSK and MQAM) and

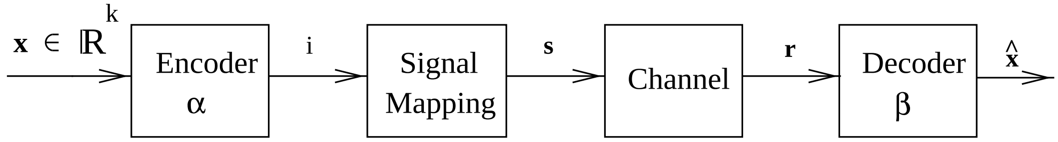

Figure 1: Block diagram of the system

two channel models (AWGN channel and Rayleigh fading channel including diversity reception techniques [7]).

2 Problem Statement

Let us consider the diagram of Figure 1 for a block source coding scheme over a waveform noisy channel.

The encoder α is described by a partition $\mathcal{P} = \{A_0, A_1, ..., A_{N-1}\}$ of $\mathbb{R}^{\mathrm{k}}$ such that

$$\alpha(\mathbf{x}) = i, \qquad \text{if} \quad \mathbf{x} \in A_i, \qquad i = 0, 1, ..., N-1 \tag{1}$$

$\mathbf{x}$ is a source vector of dimension k. The signal mapping block maps an index i to a signal $\mathbf{s}$ which will be subsequently sent over the channel. Specifically, we assume that we have an *elementary* signal constellation $\mathcal{T} = \{\mathbf{t}^0, \mathbf{t}^1, ..., \mathbf{t}^{M-1}\}$, consisting of M signals each of dimension L_1, i.e., $\mathbf{t}^j \in \mathbb{R}^{\mathrm{L}_1}$, $j = 0, 1, ..., M-1$. Let us assume that $N = M^{L_2}$. Then, the signal $\mathbf{s}$ to be sent over the channel is selected from an *expanded* signal constellation $\mathcal{S} = \mathcal{T}^{L_2}$, the L_2-fold Cartesian product of $\mathcal{T}$. We assume that the signal $\mathbf{s}(i)$ used for encoding the index i is given by

$$\mathbf{s}(i) = (\mathbf{t}^{(i_0)}, \mathbf{t}^{(i_1)}, ..., \mathbf{t}^{(i_{L_2-1})}) \tag{2}$$

where $(i_{L_2-1} i_{L_2-2} ... i_1 i_0)$ is the representation of i in base M, i.e.,

$$i = \sum_{j=0}^{L_2-1} M^j i_j \tag{3}$$

Note that the effective dimension of the expanded signal set $\mathcal{S}$ is $L = L_1 L_2$. The ratio L/k is a measure of bandwidth efficiency of the system ([5]).

In principle, we are going to consider that the channel is an additive white Gaussian noise channel. The random channel output $\mathbf{r} = (r_0, r_1, ..., r_{L-1})$ is related to the input vector $\mathbf{s} = (s_0, s_0, ..., s_{L-1})$ through

$$r_i = s_i + N_i, \qquad i = 0, 1, ..., L-1 \tag{4}$$

where $N_i' s$ are independent and identically distributed (i.i.d.) Gaussian random variables with a common pdf p_N.

The decoder β makes an estimate $\hat{\mathbf{x}}$ of the source vector $\mathbf{x}$ based on the received vector (channel output) $\mathbf{r}$. In this work we only consider hard-decision decoders, in which the decoder β makes the estimate $\hat{\mathbf{x}}$ indirectly from $\mathbf{r}$ as follow. For $j =$

$0, 1, ..., L_2 - 1$, an estimate $\hat{i}_j$ of the index i_j of the transmitted elementary signal is made based on $(r_{jL_1}, r_{jL_1+1}, ..., r_{(j+1)L_1-1})$. Then, the estimate $\hat{\mathbf{x}}$ of source vector is made based on

$$\hat{i}(\mathbf{r}) = \sum_{j=0}^{L_2-1} M^j \hat{i}_j \tag{5}$$

the estimate of the transmitted index i.

3 System optimization

In optimizing the scheme of Figure 1, the objective is to minimize the average squared-error distortion between $\mathbf{X}$ and $\hat{\mathbf{X}}$ subject to a constraint on the average transmitted energy. More precisely, for a given elementary signal set, $\mathcal{T}$, a given source dimension k and a given codebook size N, (hence a fixed bandwidth efficiency), we wish to minimize

$$D = \frac{1}{k} E(\|\mathbf{X} - \hat{\mathbf{X}}\|^2) \tag{6}$$

subject to

$$\epsilon = \frac{1}{k} E(\|\mathbf{S}\|^2) \leq \epsilon_0 \tag{7}$$

where $\mathbf{S}$ denotes the L-dimensional signal delivered to the channel.

The lack of any straightforward solution to this problem forces to an algorithmic iterative solution with two steps. First, we consider the case where the encoder α is fixed. Then, from estimation theory, the optimum decoder is given as a function of the received vector $\mathbf{r}$ as follow

$$\begin{aligned} \beta^*(\mathbf{r}) &= E(\mathbf{X}|\hat{i}(\mathbf{r})) \\ &= \sum_{i=0}^{N-1} P_i E(\mathbf{X}|i) \frac{P(\hat{i}(\mathbf{r})|i)}{P(\hat{i}(\mathbf{r}))} \end{aligned} \tag{8}$$

where $P_i = Pr(\mathbf{X} \in A_i)$ and

$$P(\hat{i}(\mathbf{r})|i) = \prod_{j=0}^{L_2-1} P(\hat{i}_j|i_j). \tag{9}$$

where i_j's and $\hat{i}_j$'s are described in (3) and (5) respectively.

Denoting the signal $\mathbf{s}(i)$ associated with the index i by $\mathbf{s}(i) = (s_{i,0}, s_{i,1}, s_{i,2}, ..., s_{i,L-1})$ and if maximum-likelihood estimation is used in obtaining $\hat{i}_j$'s, we have

$$\hat{i}_j = \underset{m}{\text{argmax}} \quad p(r_{jL_1} \ r_{jL_1+1} \ \cdots \ r_{(j+1)L_1-1}|m)$$

$$= \quad \underset{m}{\operatorname{argmax}} \prod_{n=jL_1}^{(j+1)L_1-1} p_N(r_n - s_{m,n}) \tag{10}$$

Notice that in (8), $E(\mathbf{X}|i)$ is the centroid of the encoder region A_i, denoted by $\mathbf{c}_i$. Thus, the optimum decoder for a fixed encoder is given by

$$\beta^*(\mathbf{r}) = \sum_{i=0}^{N-1} P_i \mathbf{c}_i \frac{P(\hat{i}(\mathbf{r})|i)}{P(\hat{i}(\mathbf{r}))} \tag{11}$$

Second, assuming that the decoder is fixed, we shall determine the optimum encoder subject to satisfying the energy constraint. Specifically, for resolving this constraint problem we consider the Lagrangian function given by

$$\begin{aligned} L(\mathcal{P}, \lambda) &= D + \lambda(\epsilon - \epsilon_0) \\ &= \frac{1}{k} \int_{\mathbb{R}^k} F(\mathbf{x}, \lambda) p_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \end{aligned} \tag{12}$$

where

$$F(\mathbf{x}, \lambda) = E(\|\mathbf{x} - \widehat{\mathbf{X}}\|^2 | \mathbf{X} = \mathbf{x}) + \lambda(\|\mathbf{s}(\alpha(\mathbf{x}))\|^2 - k\epsilon_0) \tag{13}$$

and λ is the Lagrange multiplier. Since $p_{\mathbf{X}}(\mathbf{x}) \geq 0$ for all $\mathbf{x}$, minimizing $L(\mathcal{P}, \lambda)$ in (12) is equivalent to minimizing $F(\mathbf{x}, \lambda)$ for all $\mathbf{x}$ in $\mathbb{R}^k$. We must mention that the Lagrange multiplier λ should be chosen so that the energy constraint (7) is satisfied. For a given λ, the optimum partition $\mathcal{P}^*$ is described by the optimum encoding regions $A_i^*(\lambda)$, $i = 0, 1, \ldots, N-1$, given by

$$A_i^*(\lambda) = \{\mathbf{x} : F_i(\mathbf{x}, \lambda) \leq F_j(\mathbf{x}, \lambda), \forall j\} \tag{14}$$

where

$$\begin{aligned} F_i(\mathbf{x}, \lambda) &= E(\|\mathbf{x} - \widehat{\mathbf{X}}\|^2 | i) + \lambda(\|\mathbf{s}(i)\|^2 - k\epsilon_0) \\ &= \|\mathbf{x}\|^2 + E(\|\widehat{\mathbf{X}}\|^2 | i) - 2\left\langle \mathbf{x}, E(\widehat{\mathbf{X}}|i) \right\rangle + \lambda(\|\mathbf{s}(i)\|^2 - k\epsilon_0) \end{aligned} \tag{15}$$

$< \cdot, \cdot >$ denotes the inner product. Upon denoting $E(\widehat{\mathbf{X}}|i)$ by $\mathbf{a}_i$ and $E(\|\widehat{\mathbf{X}}\|^2|i)$ by b_i, we can rewrite (14) in the following form

$$A_i^*(\lambda) = \left\{ \mathbf{x} : \langle \mathbf{x}, (\mathbf{a}_i - \mathbf{a}_j) \rangle \geq \frac{(b_i - b_j) + \lambda(\|\mathbf{s}(i)\|^2 - \|\mathbf{s}(j)\|^2)}{2}, \forall j \right\} \tag{16}$$

It is clear from (16) that $A_i^*(\lambda)$ is a convex polyhedron as in conventional vector quantization design ([1]). Note that the $\mathbf{a}_i$'s and b_i's only depend upon the channel and the decoder. Under the assumption of hard-decision decoding we have

$$\mathbf{a}_i = \sum_{\hat{i}=0}^{N-1} \beta(\hat{i}) P_{\hat{I}|I}(\hat{i}|i) \tag{17}$$

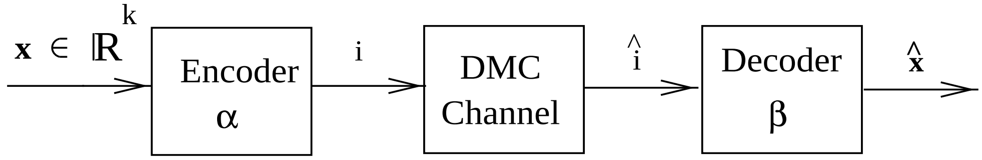

Figure 2: Block diagram of the equivalent system

and

$$b_i = \sum_{\hat{i}=0}^{N-1} \|\beta(\hat{i})\|^2 P_{\hat{I}|I}(\hat{i}|i) \tag{18}$$

where we have committed an abuse of notation by using $\beta(\hat{i})$ to denote $\beta(\hat{i}(\mathbf{r})) \equiv \beta(\mathbf{r})$.

A successive application of equations (11) and (16) results in a sequence of encoder-decoder pairs for which the corresponding average distortion form a non-increasing sequence of non-negative numbers which has to converge.

3.1 Optimization for Rayleigh fading channels

From expression (6) we get

$$D = \frac{1}{k} \sum_{i=0}^{N-1} \int_{A_i} p(\mathbf{x}) \left\{ \sum_{\hat{i}=0}^{N-1} P(\hat{i}|i) \|\mathbf{x} - \beta(\hat{i})\|^2 \right\} \mathrm{d}\mathbf{x} \tag{19}$$

where $p(\mathbf{x})$ is the k-dimensional source pdf. With this last expression (19) for the average distortion and from the condition of a hard-decision decoder, the system can be viewed (Figure 2) as an equivalent system in which the modulator (included in signal mapping module of Figure 1), the channel and the demodulator (included in the decoder of Figure 1) form a DMC. This characteristic explains the equivalency of the obtained optimum expressions of the encoder and the decoder for a Gaussian channel and those of COVQ ([4]).

If the channel is a flat-fading Rayleigh channel, $\mathbf{r}$ and $\mathbf{s}$ are related through

$$r_i = \alpha s_i + N_i, \qquad i = 0, 1, ..., L-1 \tag{20}$$

where α is the time varying gain of the channel, which is subject to fading.

For system optimizing, assuming that the channel is a flat-fading Rayleigh channel, we can consider the same situation (as the DMC channel), with the only difference that transition probabilities are in this case functions of the received SNR, γ, (the channel SNR, CSNR) defined as

$$\gamma = \alpha^2 \frac{\varepsilon_i}{N_0} \tag{21}$$

where ε_i is the energy of the transmitted signal and N_0 is the one-sided spectral density of the noise. Therefore, to compute the average distortion of the system we have to use average values of transition probabilities over all values of γ. In other words, we have to compute

$$\overline{P(j|i)} = \int_0^\infty P(j|i)p(\gamma)\mathrm{d}\gamma \quad i,j \in \mathcal{I} \tag{22}$$

where $P(j|i)$ are transition probabilities for an AWGN channel and $p(\gamma)$ is the pdf of γ given by ([8])

$$p(\gamma) = \frac{1}{\Gamma}\exp^{-\gamma/\Gamma} \qquad \gamma \geq 0 \tag{23}$$

where Γ represents the average received SNR.

To optimize the system for transmission over a flat-fading Rayleigh channel we can use the expressions as in case of AWGN channel using the average transition probabilities.

In the same way, it is possible to extend the study of the system performance to the case of using diversity technique to mitigate the effects of deep fades. In this situation, the most common used technique is antenna diversity. We consider a B-fold antenna diversity on arbitrary flat-fading Rayleigh channel. Maximum ratio combining (MRC) and selection combining (SC) are considered as diversity reception techniques. For these methods, the pdf of γ is given by

- MRC with B i.i.d. channels ([8])

$$p_{MRC}(\gamma) = \frac{1}{(B-1)!}\frac{\gamma^{B-1}}{\Gamma^B}e^{-\gamma/\Gamma} \qquad \gamma > 0 \tag{24}$$

  where $\Gamma = \mathrm{E}\{\gamma_k\}$ is the average SNR on the kth channel, which is considered to be the same over all the channels.

- SC ([9])

$$p_{SC}(\gamma) = \sum_{k=1}^{B}\binom{B}{k}(-1)^{k-1}\frac{k}{\Gamma}e^{-k\gamma/\Gamma} \qquad \gamma > 0 \tag{25}$$

  again $\Gamma = \mathrm{E}\{\gamma_k\}$ is the average SNR on the kth channel.

There exists exact expressions to compute integrals (22) for most cases of extended signal constellations with considered $p(\gamma)$'s, expressions (23), (24) and (25). They have been computed carrying out an extra analysis of the resolution of integrals computed in [8], [9] and [10].

4 Numerical Results and Discussions

In order to provided a performance evaluation of the proposed system, we consider a first order Gauss-Markov source with correlation coefficients $\rho = 0.9$. The results are given in terms of the output SNR, given by

$$SNR \equiv -10 \log_{10} D/\sigma^2 \tag{26}$$

where D is the per-sample distortion and σ^2 is the source variance, for various values of the average channel SNR.

Tables 1-7 present performance results for COVQ over waveform channels for several rates and source vector dimensions. Columns labelled as (1), (2), (3) and (4) show performance results for an AWGN channel, Rayleigh channel, Rayleigh channel with a 2-fold MRC diversity receiver and Rayleigh channel with a 2-fold SC diversity receiver, respectively.

From the obtained results it is observed that COVQ systems with QAM modulation achieve better or, at least, the same SNR performance than those with PSK modulation with the same system parameters, due to the higher probability of symbol error of PSK signal constellations. For binary signaling (binary PSK, BPSK), compared to QAM modulation performance, better results are achieved for slight or moderate noisy channel, for example, for a CNSR of 6 dB o greater in Table 4 binary signaling gives better or same results than QAM with $L = 2$. However, the binary signaling performance is not favorably compares for very noisy channel condition (less than a CNSR of 3 db in the previous example). In addition, it is observed that when the factor L/k increases the performance is better for slight or moderate noisy channel and poor for a noisy channel with respect to smaller values of L/k. For example, for a CNSR of 6 dB o greater in Table 4, QAM with $L = 4$ gives better or same results with respect to QAM with $L = 2$. However,it gives poor results for a CNSR of 3 dB or less.

From results obtained for different channel models, it is observed that there is a severe penalty in system performance for a Rayleigh channel. However, the system performance can be improved when a diversity technique reception is used. The MRC technique is favorably compared to SC technique, as expected, but with the incurred penalty of having a higher cost of implementing that technique.

5 Conclusions

In this work COVQ system is generalized for transmission over waveform channels under the assumptions of hard-decision decoding. Several modulation techniques have been studied, concluding that it is better to use QAM instead of PSK. With respect to binary signaling, this modulation technique is preferable when the channel is not very noisy. The study are applied to Rayleigh channel without and with the usage of diversity technique reception. It is shown that with a two-branch diversity reception, compared to the AWGN channel, similar performance results can be obtained.

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CSNR (dB)	PSK (L=2)				QAM (L=2)				BPSK			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
18.0	6.52	6.37	6.52	6.52	6.52	6.37	6.52	6.52	6.52	6.37	6.52	6.52
15.0	6.52	6.23	6.52	6.52	6.52	6.23	6.52	6.52	6.52	6.23	6.52	6.52
12.0	6.52	5.97	6.51	6.50	6.52	5.97	6.51	6.50	6.52	5.97	6.51	6.50
9.0	6.52	5.52	6.47	6.42	6.52	5.52	6.47	6.42	6.52	5.52	6.47	6.42
6.0	6.52	4.81	6.34	6.19	6.52	4.81	6.34	6.19	6.52	4.81	6.34	6.19
3.0	6.34	3.86	5.93	5.57	6.34	3.86	5.93	5.57	6.34	3.86	5.93	5.57
0.0	5.06	2.79	5.03	4.42	5.06	2.79	5.03	4.42	5.06	2.79	5.03	4.42
-3.0	3.04	1.80	3.66	3.00	3.04	1.80	3.66	3.00	3.04	1.80	3.66	3.00

Table 1: Output SNR for a first order Gauss-Markov source, correlation factor $\rho = 0.9$; $k = 4$; $R_s = 0.5$ bits/sample; $N = 4$

CSNR (dB)	PSK (L=4)				QAM (L=4)				PSK (L=2)				QAM (L=2)				BPSK			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
18.0	7.81				7.81	7.56	7.81	7.81	7.81	7.21	7.79	7.77	7.81	7.53	7.81	7.80	7.81	7.56	7.81	7.81
15.0	7.81				7.81	7.33	7.81	7.80	7.81	6.92	7.73	7.66	7.81	7.30	7.80	7.78	7.81	7.33	7.81	7.80
12.0	7.82	6.91			7.82	6.92	7.80	7.77	7.77	6.52	7.54	7.36	7.81	6.37	7.76	7.71	7.82	6.92	7.79	7.77
9.0	7.82	6.35	7.73	7.66	7.82	6.25	7.77	7.64	7.26	5.92	7.21	7.00	7.81	6.37	7.63	7.49	7.82	6.25	7.73	7.65
6.0	7.81	5.43	7.53	7.30	7.81	5.28	7.65	7.23	6.83	5.30	6.74	6.44	7.53	5.67	7.29	7.00	7.81	5.28	7.51	7.27
3.0	7.52	4.28	6.92	6.40	7.52	4.63	7.28	6.26	6.21	4.57	6.28	5.78	6.57	4.80	6.62	6.25	7.52	4.99	6.89	6.35
0.0	5.67	3.62	5.68	4.90	5.67	3.49	6.45	5.21	5.49	3.77	5.48	4.96	5.58	3.89	5.78	5.35	5.67	4.04	5.60	4.81
-3.0	4.37	2.77	4.59	4.04	4.37	2.44	5.29	3.72	4.71	2.92	4.66	4.15	4.71	2.98	4.84	4.37	4.37	2.97	4.87	4.26

Table 2: Output SNR for a first order Gauss-Markov source, correlation factor $\rho = 0.9$; $k = 8$; $R_s = 0.5$ bits/sample; $N = 16$

CSNR (dB)	PSK (L=2)				QAM (L=2)				BPSK			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
18.0	7.92	7.50	7.92	7.91	7.92	7.50	7.92	7.91	7.92	7.49	7.92	7.91
15.0	7.92	7.13	7.90	7.88	7.92	7.13	7.90	7.88	7.92	7.10	7.90	7.88
12.0	7.92	6.51	7.84	7.78	7.92	6.51	7.84	7.78	7.92	6.47	7.84	7.77
9.0	7.92	5.59	7.65	7.44	7.92	5.59	7.65	7.44	7.92	5.52	7.64	7.43
6.0	7.65	4.41	7.08	6.58	7.65	4.41	7.08	6.58	7.65	4.32	7.07	6.56
3.0	5.90	3.14	5.87	5.09	5.90	3.14	5.87	5.09	5.90	3.05	5.84	5.05
0.0	3.42	2.01	4.16	3.38	3.42	2.01	4.16	3.38	3.42	1.94	4.12	3.34
-3.0	1.76	1.17	2.56	1.98	1.76	1.17	2.56	1.98	1.76	1.13	2.52	1.95

Table 3: Output SNR for a first order Gauss-Markov source, correlation factor $\rho = 0.9$; $k = 2$; $R_s = 1.0$ bits/sample; $N = 4$

CSNR (dB)	PSK (L=4)				QAM (L=4)				PSK (L=2)				QAM (L=2)				BPSK			
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
18.0	10.17				10.17	9.29	10.17	10.15	10.17	8.96	10.03	9.92	10.17	9.25	10.14	10.11	10.17	9.30	10.16	10.15
15.0	10.18	8.60			10.17	8.61	10.15	10.08	10.10	8.43	9.75	9.51	10.17	8.67	10.05	9.95	10.17	8.63	10.13	10.09
12.0	10.18	7.76	10.01	9.87	10.18	7.61	10.08	9.83	9.47	7.60	9.29	9.08	10.15	7.96	9.80	9.58	10.18	7.64	10.01	9.86
9.0	10.17	6.46	9.62	9.23	10.17	6.25	9.85	9.12	8.74	6.72	8.81	8.43	9.64	7.02	9.23	8.80	10.17	6.30	9.60	9.19
6.0	9.62	5.65	8.63	7.84	9.62	5.55	9.21	7.63	8.11	5.79	8.08	7.64	8.35	5.93	8.38	7.91	9.62	6.13	8.58	7.78
3.0	6.81	4.30	6.81	6.50	6.81	4.14	7.91	6.31	7.25	4.71	7.06	6.61	7.24	4.80	7.31	6.84	6.81	4.87	6.73	6.74
0.0	5.27	3.22	5.71	4.93	5.27	3.19	6.53	4.48	6.06	3.55	5.99	5.36	6.01	3.61	6.13	5.48	5.27	3.50	5.94	5.18
-3.0	3.24	2.13	4.05	3.34	3.24	2.22	5.35	3.30	4.25	2.57	4.59	3.96	4.40	2.51	4.70	4.04	3.24	2.27	4.25	3.51

Table 4: Output SNR for a first order Gauss-Markov source, correlation factor $\rho = 0.9$; $k = 4$; $R_s = 1.0$ bits/sample; $N = 16$

CSNR (dB)	PSK (L=8) (1)	(2)	(3)	(4)	QAM (L=8) (1)	(2)	(3)	(4)	PSK (L=4) (1)	(2)	(3)	(4)	QAM (L=4) (1)	(2)	(3)	(4)
18.0	11.43				11.43	10.23	11.42	11.39	11.43				11.43	10.12	11.41	11.33
15.0	11.44				11.43	9.38	11.40	11.29	11.29				11.43	9.68	11.34	11.09
12.0	11.44	8.63	10.57		11.44	8.60	11.29	10.91	10.62	8.56	9.98		11.40	9.12	11.11	10.51
9.0	11.42	7.80	10.60	10.12	11.43	7.53	10.94	9.92	9.90	7.67	9.96	9.60	10.65	8.23	10.58	9.97
6.0	10.63	6.63	9.57	9.09	10.62	6.28	10.05	8.74	9.20	6.64	9.15	8.69	9.52	7.19	10.17	9.21
3.0	8.66	5.27	8.42	7.58	8.66	4.92	8.96	7.14	8.20	5.49	8.07	7.53	8.29	6.08	9.49	8.14
0.0	6.31	3.93	6.65	5.85	6.32	3.79	7.58	5.41	6.91	4.33	6.79	6.13	6.97	4.86	8.52	6.86
-3.0	4.24	2.80	4.93	4.17	4.25	2.75	6.13	4.00	5.01	3.21	5.41	4.68	5.41	3.63	7.42	5.41

CSNR (dB)	PSK (L=2) (1)	(2)	(3)	(4)	QAM (L=2) (1)	(2)	(3)	(4)	BPSK (1)	(2)	(3)	(4)
18.0	9.42	8.33	9.47	9.27	10.94	9.69	10.75	10.65	11.43	10.17	11.41	11.40
15.0	9.00	7.73	9.03	8.80	10.39	9.05	10.49	10.24	11.43	9.31	11.36	11.30
12.0	8.50	7.10	8.53	8.25	9.71	8.32	9.94	9.60	11.44	9.13	11.18	10.96
9.0	7.90	6.48	7.91	7.61	9.12	7.44	9.23	8.87	11.43	8.34	10.59	10.04
6.0	7.30	5.68	7.28	7.00	8.39	6.55	8.52	8.09	10.62	7.22	9.78	9.30
3.0	6.62	4.81	6.59	6.21	7.59	5.61	7.70	7.23	8.66	5.95	8.74	7.97
0.0	5.81	3.92	5.79	5.37	6.59	4.54	6.74	6.20	6.33	4.53	7.08	6.31
-3.0	4.82	3.05	4.84	4.37	5.46	3.47	5.73	5.04	4.25	3.21	5.37	4.56

Table 5: Output SNR for a first order Gauss-Markov source, correlation factor $\rho = 0.9$; $k = 8$; $R_s = 1.0$ bits/sample; $N = 256$

CSNR (dB)	PSK (L=2) (1)	(2)	(3)	(4)	QAM (L=2) (1)	(2)	(3)	(4)	BPSK (1)	(2)	(3)	(4)
18.0	13.36	10.66	12.57	12.05	13.54	10.75	13.30	13.11	13.54	10.63	13.44	13.35
15.0	11.76	9.49	11.38	11.89	13.50	9.77	12.80	12.34	13.55	9.07	13.17	12.87
12.0	11.01	8.25	10.93	10.87	12.45	8.47	11.71	11.03	13.53	7.24	12.36	11.60
9.0	10.07	6.83	9.89	9.57	10.50	7.00	10.45	9.81	12.38	7.09	10.52	9.27
6.0	8.73	5.45	8.47	8.04	8.73	5.51	8.96	8.15	7.91	5.57	7.81	6.47
3.0	7.02	4.10	6.94	6.30	7.14	4.08	7.21	6.38	6.05	3.95	6.91	5.93
0.0	4.74	2.85	5.18	4.46	5.06	2.81	5.43	4.59	3.65	2.54	4.83	3.96
-3.0	2.89	1.89	3.57	3.00	3.09	1.80	3.70	3.02	2.05	1.51	3.02	2.39

Table 6: Output SNR for a first order Gauss-Markov source, correlation factor $\rho = 0.9$; $k = 2$; $R_s = 2.0$ bits/sample; $N = 16$

CSNR (dB)	PSK (L=4) (1)	(2)	(3)	(4)	QAM (L=4) (1)	(2)	(3)	(4)	PSK (L=2) (1)	(2)	(3)	(4)	QAM (L=2) (1)	(2)	(3)	(4)	BPSK (1)	(2)	(3)	(4)
18.0	15.42	9.57			15.75	12.55	15.49	14.92	11.79	10.02	11.79	11.45	13.81	11.43	14.01	13.62	15.75	12.61	15.56	15.38
15.0	14.29	8.10			15.66	11.42	14.96	13.81	11.07	9.10	11.06	10.72	12.71	10.34	13.07	12.56	15.75	11.75	15.08	14.56
12.0	13.11	6.63	13.10	12.47	14.07	10.09	13.91	13.03	10.29	8.20	10.31	9.97	11.82	9.23	12.04	11.51	15.72	10.51	13.77	12.71
9.0	11.94	5.13	11.73	11.03	12.14	8.63	13.28	11.87	9.48	7.13	9.49	9.01	10.93	8.01	10.97	10.36	13.79	8.92	12.58	11.86
6.0	10.50	3.74	10.19	9.44	10.41	7.16	12.11	10.53	8.54	6.01	8.49	8.05	9.77	6.73	9.75	9.08	10.79	7.24	11.00	9.96
3.0	8.64	2.58	8.52	7.61	8.68	5.77	10.66	8.69	7.56	4.86	7.44	6.87	8.34	5.43	8.56	7.65	7.71	5.45	8.77	7.66
0.0	6.15	1.66	6.61	5.72	6.54	4.24	8.85	6.47	6.22	3.70	6.19	5.53	6.93	4.17	7.08	6.32	5.13	3.71	6.49	5.34
-3.0	3.91	1.01	4.75	3.98	4.45	3.04	7.27	4.78	4.33	2.62	4.74	4.06	5.30	2.99	5.50	4.77	3.23	2.43	4.29	3.31

Table 7: Output SNR for a first order Gauss-Markov source, correlation factor $\rho = 0.9$; $k = 4$; $R_s = 2.0$ bits/sample; $N = 256$